Noise effects in the ac-driven Frenkel-Kontorova model

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The noise effects on dynamical-mode-locking phenomena in the ac-driven dissipative Frenkel-Kontorova model are studied by molecular-dynamics simulations. It was found that the noise strongly influences the properties of the Shapiro steps and the way they respond to the changing of system parameters. The increase of temperature produces the melting of the Shapiro steps, while the critical depinning force is significantly reduced. The oscillatory form of the amplitude dependence is strongly affected where the Bessel-like form changes as the temperature increases. In the frequency dependence of the Shapiro steps, due to the decrease of the dc threshold value, noise may transfer the system to the high-amplitude regime where oscillations of the step width with frequency or period of the ac force appear. These phenomena will additionally destabilize the steps in real systems and significantly limit the region of parameters where dynamical-mode-locking phenomena could be observed.

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I. INTRODUCTION

In recent years, due to possible technical applications of interference effects, the influence of noise on dynamical mode-locking phenomena has been the subject of extensive theoretical and experimental studies in systems such as charge-density-wave conductors [1–3] and systems of Josephson-junction arrays biased by external currents [4–10]. Numerous experimental and theoretical results and the great complexity of these dissipative many-body systems have stimulated studies of the dissipative (overdamped) Frenkel-Kontorova (FK) model as one of the simplest among many-body models, but still complex enough that it can capture the essence of many physical phenomena. Motivated by the great significance of the noise problem for experiments and technical applications, in this work, we will study the noise effects on the (dc+ac)-driven overdamped FK model.

The one-dimensional standard FK model represents a chain of harmonically interacting particles subjected to a sinusoidal substrate potential [11]. It describes different commensurate or incommensurate structures that show very rich dynamical behavior under an external driving force. While extensive studies have been performed on the dc-driven FK model, a relatively small number of studies have been dedicated to the FK model driven by periodic forces [12]. The dynamics of the (dc+ac)-driven FK model is characterized by the appearance of the staircase macroscopic response or Shapiro steps in the curve for average velocity as a function of the average external driving force $\bar{v}(\bar{F})$ [12]. These steps are due to interference or dynamical mode-locking of the internal frequency (which comes from the motion of particles over periodic substrate potential) with the frequency of an external ac force [12,13]. Dynamical mode locking is only possible if the set of ground states is discrete and appears to be one of the universal features of the systems with the competition of time scales in the ac-driven dynamics.

In the present paper, we will examine how thermal noise affects the dynamics of the commensurate structures in the (dc+ac)-driven overdamped FK model—in particular, how it affects the existence and the properties of the Shapiro steps (the incommensurate structures that are characterized by the dynamical Aubry transition [12] represent a different problem and will be part of our future examinations). We have been not only interested in how the step size and the critical depinning force change, but also how the noise influences the amplitude and frequency dependence of the steps [14]. It was shown that besides the melting of the Shapiro steps and strong decrease of the critical depinning force, noise can completely change the properties of the steps and the way they respond to the changing of system parameters. While the well-known Bessel-like oscillations with amplitude change their form under noise, in the case of frequency dependence, noise may transfer the system to the high-amplitude regime where oscillations of the step size with frequency appear. In our previous studies of the standard FK model ($T=0$) [14], we have shown that oscillations of the step width with frequency appear if the system is in the high-amplitude regime $F_{ac}>F_{c0}$ (the applied ac amplitude is larger than the dc threshold). The fact that these oscillations have been observed in the standard FK model raises a serious question, whether they are only a peculiarity of the FK model or they could exist and be relevant in real systems. Our studies of the same phenomena in a realistic system have shown that these oscillations of the step width with frequency may not only exist in real systems, but environmental effects such as noise can even contribute to their appearance [14]. Here we will present additional results obtained in the presence of noise that prove the existence and universality of these phenomena and their importance for experiments and technical applications of the interference effects.

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The paper is organized as follows. The model is introduced in Sec. II. Simulation results are presented and analyzed in Sec. III, where the influence of noise on the amplitude dependence is discussed in Sec. III A and on the frequency dependence in Sec. III B. Finally, Sec. IV concludes the paper.

II. MODEL

We consider the dissipative (overdamped) dynamics of a series of coupled harmonics oscillators u_l subjected in a sinusoidal substrate (pinning) potential:

$$V(u) = \frac{K}{(2\pi)^2}[1-\cos(2\pi u)], \tag{1}$$

where K is the pinning strength. The system is driven by dc and ac forces:

$$F(t) = \bar{F} + F_{ac}\cos(2\pi\nu_0 t). \tag{2}$$

The equation of motions is

$$\dot{u}_l = u_{l+1} + u_{l-1} - 2u_l - V'(u_l) + F(t) + L_l(t), \tag{3}$$

where $l=-\frac{N}{2},\ldots,\frac{N}{2}$ and the thermal noise satisfies $\langle L_l(t)L_l(t')\rangle = 2T\delta(t-t')$.

When the system is driven by homogenous periodic force, the competition between two frequency scales (the frequency ν_0 of the external periodic ac force and the characteristic frequency of the motion over the periodic substrate potential driven by the average force $\bar{F}$) can result in the appearance of synchronization phenomena (resonance). If $u_l(t)$ is the solution of Eq. (3), then the transformation

$$\sigma_{i,j,m}\{u_l(t)\} = \{u_{l+i}(t - m/\nu_0) + j\} \tag{4}$$

produces another solution, where i, j, and m are integers. The solution is called resonant if there is a triplet of integers such that it is invariant under the symmetry operation as follows:

$$\sigma_{i,j,m}\{u_l(t)\} = \{u_l(t)\}. \tag{5}$$

The average velocity of the resonant solution is given by [12]

$$\bar{v} = \frac{i\omega + j}{m}\nu_0, \tag{6}$$

where the interparticle average distance (winding number) $\omega = \langle (u_{l+1} - u_l)\rangle$ (ω is rational for the commensurate and irrational for the incommensurate structures). When $m=1$, the resonant solutions and the steps are called harmonic, while when $m>1$, the steps are called subharmonic ($m=2$ for fractional or half-integer steps).

Equation (3) has been integrated using periodic boundary conditions for the commensurate structure $\omega=\frac{1}{2}$ (two particles per potential well). The time step used in the simulations was 0.001τ, and a time interval of 100τ was used as a relaxation time to allow the system to reach the steady state (system size and commensurability effects have been tested; they are important only at very high temperatures $T>1$). The force was varied with step 10^{-4}. The response function $\bar{v}(\bar{F})$—in particular, the step width and the critical depinning force—is analyzed for different amplitudes and frequencies of the ac force, at different levels of noise (the system is considered to be on the step if the changes of $\bar{v}$ are less than 0.1%).

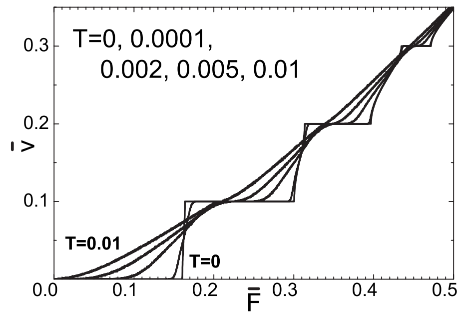

FIG. 1. Average velocity as a function of the average driving force for $\omega=\frac{1}{2}$, $K=4$, $F_{ac}=0.2$, and $\nu_0=0.2$ and different values of the temperature $T=0$, 0.0001, 0.002, 0.005, and 0.01.

III. RESULTS

In (dc+ac)-driven systems, the presence of the ac force induces additional polarization energy into the system that is different from zero (less than zero) only when the velocity reaches the resonant values, while at the same time, the average pinning force will also be different from zero. The system will get locked since the average pinning energy of the locked state (on the step) is lower than in the unlocked state. As $\bar{F}$ increases, the particles will stay locked until the pinning force can cancel the changes of $\bar{F}$. However, the presence of thermal noise will bring an additional contribution to the energy of particles and, therefore, strongly affect the mode-locking and the stability of the steps. The existence and robustness (structural stability) of the resonant solutions are always the main focus in the examination of interference phenomena.

In Fig. 1, the response function $\bar{v}(\bar{F})$ for the commensurate structure $\omega=\frac{1}{2}$ is presented for different values of the temperature.

As T increases, the Shapiro steps start to melt, becoming more and more rounded, and completely disappear; meanwhile, the critical depinning force F_c also decreases. At high temperature, the pinning potential can be neglected and the system behaves as a system of free particles. Melting is not the only effect that noise induces; it also changes the properties of the Shapiro steps and their behavior towards the changing system parameters. Further, we will present a detailed analysis of the amplitude and frequency dependence of the step width and the critical depinning force in the presence of noise. We will consider only the behavior of the harmonic steps, since in the standard FK model, subharmonic steps appear only for rational noninteger values of ω. However, even at $T=0$ their size is too small [12], and at any temperature different from zero they disappear. Large subharmonic steps can appear in the nonstandard FK model, such as one with an asymmetric deformable substrate poten-

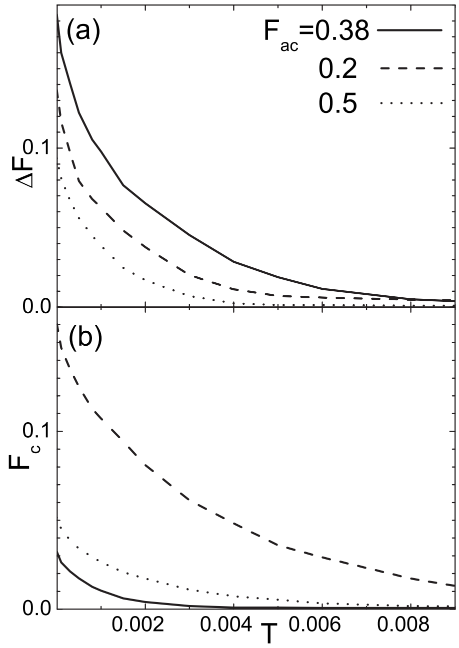

FIG. 2. The width ΔF of the first harmonic step in (a) and the critical depinning force F_c in (b) as functions of temperature for $\omega=\frac{1}{2}$, $K=4$, $\nu_0=0.2$, and $F_{ac}=0.2$, 0.38, and 0.5.

tial [15]. There, subharmonic steps appear as a result of the deformation of substrate potential.

A. Amplitude dependence of Shapiro steps in the presence of noise

In Figs. 2(a) and 2(b), the decrease of the step width ΔF for the first harmonic ($\bar{v}=\frac{1}{1}\omega\nu_0$) and the critical depinning force F_c with the increase of temperature for different values of amplitude are presented.

As we can see, with the increase of T, the step width and the critical depinning force decrease to zero (which is in agreement with experiments [4,5]), where the reduction strongly depends on the amplitude of the ac force. It is well known that ΔF and F_c exhibit Bessel-like oscillations with the ac amplitude [2,14,19–21], where the maxima of one curve correspond to the minima of another. Therefore, in Fig. 2(a), the step size has the largest value for $F_{ac}=0.38$, which corresponds to the maximum step size (the first maximum of the Bessel function); meanwhile, in Fig. 2(b), this value of the ac amplitude corresponds to the lowest curve. During the examination, we have also observed that the increase of F_{ac} reduces the rounding of the steps due to the noise.

Variations of the step width for the first harmonic and the critical depinning force with the ac amplitude at different values of T are shown in Figs. 3(a) and 3(b), respectively.

The step width and the critical depinning force oscillate at all temperatures; however, the Bessel form of oscillations completely changes due to the noise.

These Bessel-like oscillations of the step size with amplitude appear due to back and forward displacement of particles induced by the ac force. Namely, in (dc+ac)-driven systems, the dynamics is characterized by a combination of the two types of motions: linear motion in the direction of the dc force and backward and forward jumps due to the ac force. Therefore, due to the presence of dc and ac forces, the particles perform motion that consists of a series of backward and forward jumps, where the ac amplitude determines how much this motion is retarded [2]. In Fig. 4, the motion of one particle during one period of ac force is presented. If we consider a particle at the site i, then during one period, a particle will first jump n sites backward, reach the $i-n$ site, and then hop again $n+1$ sites forward to the site $i+1$. During the next period, it will repeat again these backward and forward jumps and move to the site $i+2$. In that way, by repeating these backward and forward jumps with every period of the ac force it will move. The distance (the number of sites,

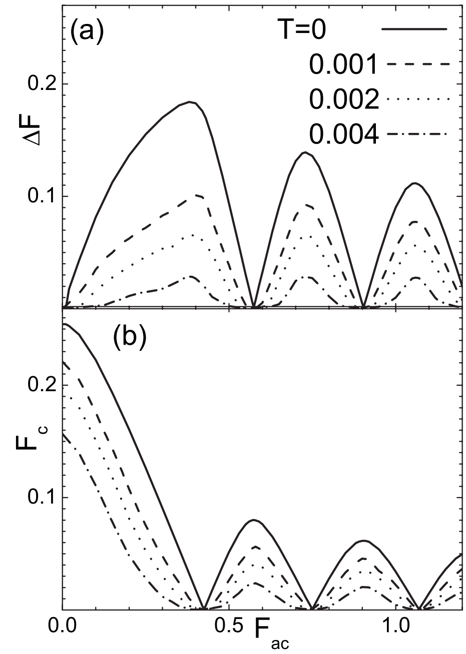

FIG. 3. The width ΔF of the first harmonic step in (a) and the critical depinning force F_c in (b) as functions of the ac amplitude for $\omega=\frac{1}{2}$, $K=4$, $\nu_0=0.2$, and $T=0$, 0.001, 0.002, and 0.004.

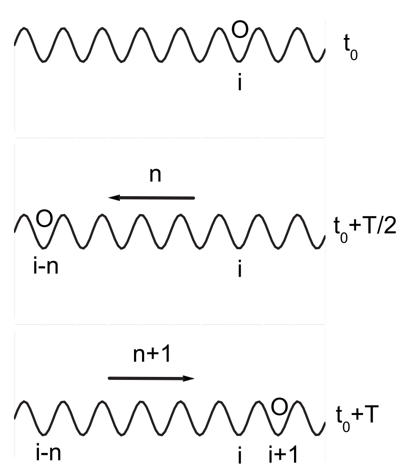

FIG. 4. The motion of a particle during one period of the ac force, where $n=0,1,2,\ldots$. is the number of sites over which the particle moves.

n) over which particles moves during these backward and forward jumps is determined by the amplitude of the ac force [2]. For the values of the ac amplitude that correspond to the first maximum in Fig. 3(a), particles will spend most of the time on the site and then hop to the next well, while for the values at the second maximum, particles will jump one site backward and two forward. As the ac amplitude increases, the particles will hop between wells that are more and more distant while spending less time on the sites, and consequently, the step width will decrease. If the noise is present, the behavior will completely change due to the additional contribution to the energy of the particles. While at $T=0$ the maxima of the Bessel function decrease as the number of sites between which particles move increases, in the presence of noise, due to the additional energy contribution, even for the value of F_{ac} that corresponds to the first maximum, particles will have possibility to move between more distant sites, and consequently, the first maxima will be significantly reduced while the Bessel-like form will disappear.

In Fig. 3(a), we can clearly see that the noise induces a rounding of the minima and significant reduction of the maxima. The largest reduction of the step size is on the first maximum, and as T increases, all maxima become of the same height. Similar also happens in Fig. 3(b), where at $F_{ac}=0$, we can see the reduction of the dc threshold limit F_{c0}. The rounding of steps in Fig. 1 and the rounding of minima and the reduction of maxima (respect to the theoretical values) in Fig. 3 have been always observed in experiments, and the origins of these effects have been often a matter of discussion in charge-density-wave systems (the well-known single-particle model in charge-density-wave systems does not predict a rounding of the minima of the Bessel oscillations with amplitude) [2] and systems of Josephson junction arrays [10]. Our results show that noise is one of the factors that could be responsible for these effects.

B. Frequency dependence of Shapiro steps in the presence of noise

A decrease of the step width ΔF for the first harmonic and the critical depinning force F_c with an increase of temperature for different values of the frequency is presented in Figs. 5(a) and 5(b).

It was shown previously that in the standard FK model, ΔF increases with frequency, reaching its maximum, and then slowly decreases to zero; meanwhile, F_c increases and saturates to a frequency-independent threshold value at high frequencies [14]. Therefore in Fig. 5(a), ΔF has the largest value for $\nu_0=0.25$, which corresponds to the maximum step width, while in Fig. 5(b), we obtained a family of curves for F_c that increases as the frequency increases.

The first harmonic step width ΔF as a function of frequency in Figs. 6(a) and 6(b) and as a function of period in Figs. 6(c) and 6(d) at two different values of temperature is presented.

As we can see in Figs. 6(a) and 6(b), the presence of noise not only results in a strong reduction of ΔF, but completely changes the behavior of the steps by inducing nonmonotonic increase at low frequencies.

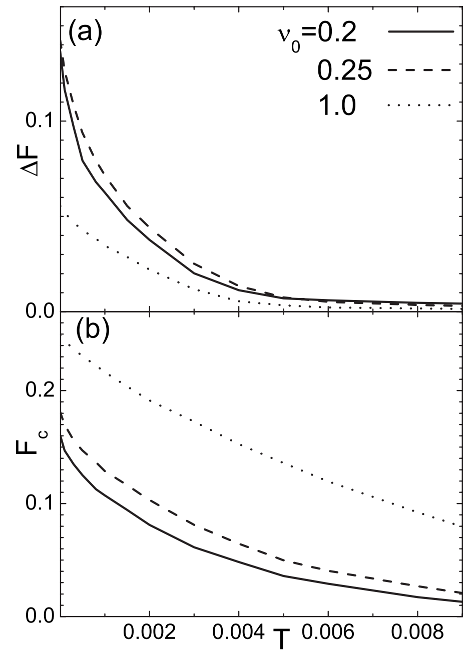

FIG. 5. The width ΔF of the first harmonic step in (a) and the critical depinning force F_c in (b) as functions of temperature for $\omega=\frac{1}{2}$, $K=4$, $F_{ac}=0.2$, and $\nu_0=0.2$, 0.25, and 1.

These low-frequency oscillations are even better revealed and their physical origin understood if the step width is plotted as a function of period $(\frac{1}{\nu_0})$, in Figs. 6(c) and 6(d). With an increase of T, the dc threshold F_{c0} decreases. Therefore, the system may change from the low-amplitude regime $F_{ac} \leqslant F_{c0}$ to the high one $F_{ac} > F_{c0}$. For the case in Figs. 6(a) and 6(c), at $T=0$, $F_{ac} \leqslant F_{c0}$ since the system is driven by the ac force with amplitude $F_{ac}=F_{c0}=0.2544$. However, at $T=0.004$ in (b) and (d), the dc threshold decreases to $F_{c0}=0.1567$, and the system is now in the high-amplitude regime $F_{ac} > F_{c0}$.

Appearance of low-frequency oscillations when $F_{ac} > F_{c0}$ is the result of the simultaneous competition and contributions of the dc and ac components of $F(t)$ to the pinning

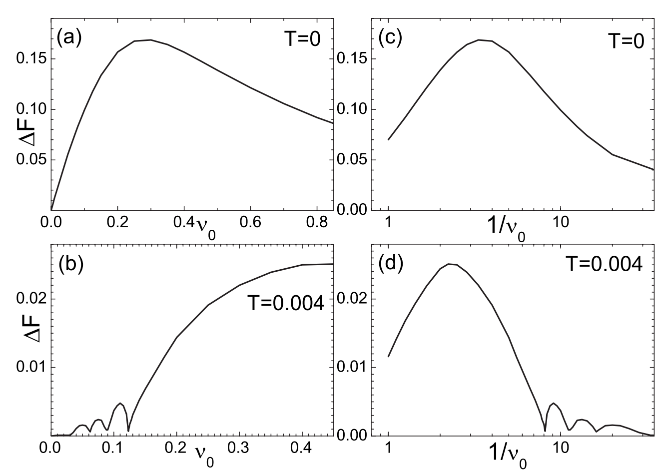

FIG. 6. The width ΔF of the first harmonic step as a function of frequency ν_0 in (a) and (b) and as a function of period $\frac{1}{\nu_0}$ in (c) and (d) for $\omega=\frac{1}{2}$, $K=4$, $F_{ac}=F_{c0}=0.2544$, and $T=0$ and 0.004.

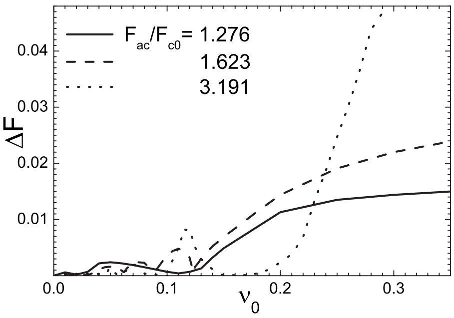

FIG. 7. The width ΔF of the first harmonic step as a function of ν_0, for $\omega=\frac{1}{2}$, $K=4$, $T=0.004$, $F_{c0}=0.1567$, and $F_{ac}=0.2$, 0.2544, and 0.5.

energy. When $\frac{F_{ac}}{F_{c0}}>1$, the ac contribution, which is responsible for the appearance of these oscillations, will dominate. In the same way as in the case of Bessel-like oscillations with amplitude in Sec. III A, these oscillations with period appear due to the backward and forward motion of particles induced by the ac force (presented in Fig. 4), where not only the ac amplitude, but also the period (frequency) determines how much this motion is retarded. Therefore, as in the case of amplitude dependence, for values of the period that correspond to the first maximum in Fig. 6(d), particles will spend most of the time on the site and then hop to the next well, while for values at the second maximum, particles will jump one site back and two forward. As the period increases, the particles will hop between the wells that are more and more distant while staying less and less time on the sites, and consequently, the step width will decrease.

If the ratio $\frac{F_{ac}}{F_{c0}}$ increases, the oscillations will spread more towards higher frequencies, while the maxima will increase, and for $\frac{F_{ac}}{F_{c0}}\gg 1$, the oscillatory behavior will dominate. The step width ΔF of the first harmonic as a function of ν_0 for three different values of $\frac{F_{ac}}{F_{c0}}$ is presented in Fig. 7.

We can clearly see that the oscillations are moving to the higher frequencies with much higher and more pronounced maxima as F_{ac} increases. For the lowest value of $\frac{F_{ac}}{F_{c0}}$, the steps are unstable for $\nu_0<0.11$, while for the highest one, they are unstable for $\nu_0<0.18$.

The step width of the first harmonic and the critical depinning force as the functions of $\frac{1}{\nu_0}$ at four different values of temperature are presented in Figs. 8(a) and 8(b), respectively.

The system is driven by the ac force with amplitude $F_{ac}=0.5$, and at $T=0$, the system is already in the high-amplitude regime $F_{ac}>F_{c0}$. These results in Fig. 8 compared with the results for the amplitude dependence in Fig. 3 clearly reveal an analogy between the amplitude and the period of the ac force. Not only that, the oscillations have a form very similar to the Bessel-like oscillations of the step width with amplitude where maxima of ΔF curves correspond to the minima of F_c curves, but they will change due to the noise in the same way as in the case of amplitude dependence. Noise will produce the rounding of minima and a significant reduction of the maxima where the Bessel-like form will be changed. These results prove again that the increase of the period has a similar effect on the backward and forward motion of particles and, therefore, on the step size as the increase of amplitude. A displacement between more distant sites will appear only if the amplitude is high enough or the period is long enough.

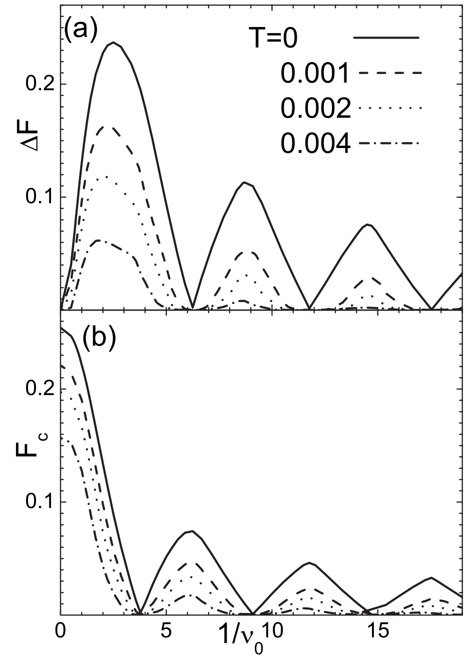

FIG. 8. The width ΔF of the first harmonic step and the critical depinning force F_c as functions of $\frac{1}{\nu_0}$, for $\omega=\frac{1}{2}$, $K=4$, $F_{ac}=0.5$, and $T=0$, 0.001, 0.002, and 0.004.

It is interesting to note that the conclusion that the ratio $\frac{F_{ac}}{F_{c0}}$ must play an important role in ac-driven dynamics can be made even intuitively without any calculation or numerical analysis by simply analyzing the driving force given in Eq. (2). In the zero-frequency limit the driving force has the form

$$F(\nu_0 \to 0) = \bar{F} + F_{ac}. \tag{7}$$

If F_{c0} is the dynamical dc threshold, then the critical depinning force at $\nu_0=0$ or the value of $\bar{F}$ at which the particles depin is given by

$$F_c(\nu_0 \to 0) = F_{c0} - F_{ac}. \tag{8}$$

This can be clearly seen from the numerical results in our previous works [14]. However, this is correct only if $F_{ac}\leqslant F_{c0}$, which naturally raises the question what will happen if $F_{ac}>F_{c0}$ and, from there, the conclusion that the point $F_{ac}=F_{c0}$ must be of some importance. The value $\frac{F_{ac}}{F_{c0}}=1$ is exactly the point when the frequency dependence of the system will change and the oscillatory behavior will appear if $F_{ac}>F_{c0}$.

The appearance of these oscillations may create additional problems in the experiments or technical applications of the interference phenomena. It is well known that the steps are less rounded and better defined if the ac amplitude increases. However, an increase of F_{ac} in order to tackle the rounding of steps due to noise while simultaneously F_{c0} is getting even reduced could have a completely contrary effect if the ratio reaches the value where $\frac{F_{ac}}{F_{c0}} \geqslant 1$, in which case, the oscillations of the Shapiro steps will become pronounced and spread even to high frequencies. Therefore, in real systems, all factors that may significantly change the ratio $\frac{F_{ac}}{F_{c0}}$ (other factors such as deformations and impurities may change F_{c0}) must be taken into account, and the parameter of the system should be adjusted in a way that $\frac{F_{ac}}{F_{c0}}$ is either smaller or around 1; otherwise, the instability region with oscillations will spread to higher frequencies.

The frequency dependence of Shapiro steps, particularly the importance of degrees of freedom, has been a matter of many controversies. In charge-density-wave (CDW) systems, two competing and fundamentally different theories have been proposed. According to the classical approach [16,17], which considers a deformable charge elastic medium with internal degrees of freedom, the step width and the critical depinning force should be strongly frequency dependent and decrease to zero at high frequencies. In contrast, in the theoretical approach based on tunneling theory [2], where the CDW conductor is treated as a macroscopic quantum system, tunneling of the CDW between the pinned states results in a frequency-independent mode locking at high frequencies. According to a simple single coordinate model motivated by tunneling theory [2], it was proven analytically (in the high-frequency limit) and experimentally that the maximum step width is proportional to the magnitude of the fundamental component of the effective pinning force that is independent of frequency at high frequencies. In systems with Josephson-junction arrays, according to the single-junction model [18–20], the width of harmonic steps remains frequency independent at high frequencies. On the other side, in models with many degrees of freedom [21,22], an amplitude and frequency dependence significantly different from the single-junction case and the disappearance of steps at high frequencies have been observed (single-junction models do not work well if the system is disordered [22]). The overdamped FK model is a classical many-body model, and as in other systems with many degrees of freedom, the steps will remain strongly frequency dependent and disappear at high frequencies. The fact that the oscillatory dependence with frequency has been observed in the FK model consequently raises the question how this phenomenon is related to the presence of many degrees of freedom in the systems. In order to examine whether these frequency oscillations appear in single-degree-of-freedom systems, we have also analyzed the commensurate structure with winding number $\omega=1$, for which the FK model reduces to the single-particle model at $T=0$ [12]. As was shown in our previous work at $T=0$ [14] and any temperature for which Shapiro steps exist, we have observed these oscillations in any commensurate structure always when $\frac{F_{ac}}{F_{c0}}>1$ and irrespectively of the number of degrees of freedom.

The important question that arises from all our studies of this phenomenon is whether these oscillations with frequency (period) could be observed in experiments. The Bessel-like oscillations with amplitude have been studied in many experiments in CDW systems [2] and the system of Josephson-junction arrays [5,6]. If the period plays the same role as the amplitude in the ac-driven dynamics, we believe that then, in the same experiments where Bessel-like oscillations with amplitude have been observed, the oscillations with the period should be also observable (especially if $\frac{F_{ac}}{F_{c0}} \geqslant 1$). The CDW systems could be a particularly good candidate since the physics behind the interference phenomena is similar to the one presented here (the similar physical picture that is presented in Sec. III A and in Fig. 4 is used to explain the amplitude dependence of the charge density waves [2]).

IV. CONCLUSION

In this paper we have presented a detailed study of the noise effects on the dynamical mode-locking phenomena in the ac-driven overdamped Frenkel-Kontorova model. The presented results have shown that the noise has a strong impact on the interference phenomena where besides producing the melting of the Shapiro steps and the reduction of the critical depinning force, it also influences the amplitude and frequency dependence of the Shapiro steps and the way they respond to the changing of the system parameters. Although the steps maintain an oscillatory dependence of the amplitude in the presence of noise, the well-known Bessel-like form of oscillations is completely changed. The most interesting is the influence of noise on the frequency dependence. By decreasing the dc threshold value, noise can transfer the system to the high-amplitude regime, and by that, it can induce oscillations of the step width with frequency. This interesting phenomenon will appear always when the ratio reaches the value $\frac{F_{ac}}{F_{c0}}>1$ in any commensurate structure and irrespectively of the number of degrees of freedom. Analyzing the step width as a function of period in the presence of noise again confirms the analogy between the amplitude and the period of the ac force in the ac-driven dynamics.

The presented results could be of great importance for all real systems with overdamped motion and driven by periodic forces such as charge- or spin-density-wave systems [1,3,23], vortex lattices [24,25], and the systems of Josephson-junction arrays [4–10]. The phenomena of the CDW in solids, which account for the anomalous transport properties, and the studies of Josephson-junction arrays are closely related to the dissipative dynamics of the FK model [1,12]. Any technical application of the interference phenomena and the building of Shapiro-step devices [7] requires a theoretical guideline for the observation of Shapiro steps where in the understanding of environmental effects, the most important one is certainly the noise effect. The phenomena that we have observed are directly related to the existence and stability of resonant solutions in real systems, which is crucial in any application of interference effects.

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